Lec 18

→ state-space representation:-

$$u(t) \rightarrow \boxed{\text{system}} \rightarrow y(t)$$

Mathematical model

① Differential equation:-

$$y^{(n)}(t) + a_{n-1} y^{(n-1)}(t) + \cdots + a_0 = b_m u^{(m)}(t) + \cdots\cdots + b_0 u(t)$$

② Transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$n \rightarrow$ system order

③ block diagram. ④ signal flow graph.

⑤ state-space model. SISO

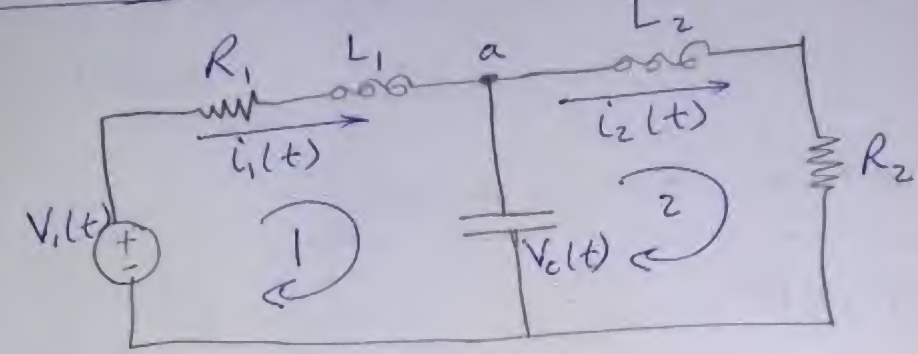
$$\dot{x}(t)_{n*1} = A_{n*n}\, x(t)_{n*1} + B_{n*1}\, u(t)_{1*1}$$

$$y(t)_{1*1} = C_{1*n} \cdot x(t)_{n*1} + D_{1*1}\, u(t)_{1*1}$$

$n \rightarrow$ number of states variable.

$\boxed{Ex}$ Find state space model

3^n order system



$X_1(t) = i_1(t)$

$X_2(t) = i_2(t)$

$X_3(t) = V_c(t)$

$V_i(t) \rightarrow \boxed{i_1(t), i_2(t), V_c(t)} \rightarrow i_2(t)$

$$\begin{bmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \\ \dot{X}_3(t) \end{bmatrix} = \begin{bmatrix} \dots & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{bmatrix} + D\,u(t)$$

$\dot{X}_1(t) = \frac{d}{dt} i_1(t)$

KVL at loop (1)

$V_i(t) = R_1 i_1(t) + L_1 \frac{d}{dt} i_1(t) + V_c(t)$

$$\boxed{\dot{X}_1(t) = \frac{-R_1}{L_1} X_1(t) - \frac{1}{L} X_3(t) + \frac{1}{L_1} u(t)} \longrightarrow (1)$$

$\dot{X}_2(t) = \frac{d}{dt} i_2(t)$

→ Apply KVL at loop 2 :-

$V_c(t) + L_2 \frac{d}{dt} i_2(t) + i_2 R_2 = 0$

$$\boxed{\dot{X}_2(t) = \frac{-R_2}{L_2} X_2(t) - \frac{1}{L_2} X_3(t)} \longrightarrow (2)$$

$\dot{X}_3(t) = \frac{d}{dt} V_c(t)$

* apply KCL at node (a)

$i_1(t) = C \frac{dV_c(t)}{dt} + i_2(t)$

$$\dot{X}_3(t) = \frac{1}{C} X_1(t) - \frac{1}{C} X_2(t) \longrightarrow (3)$$

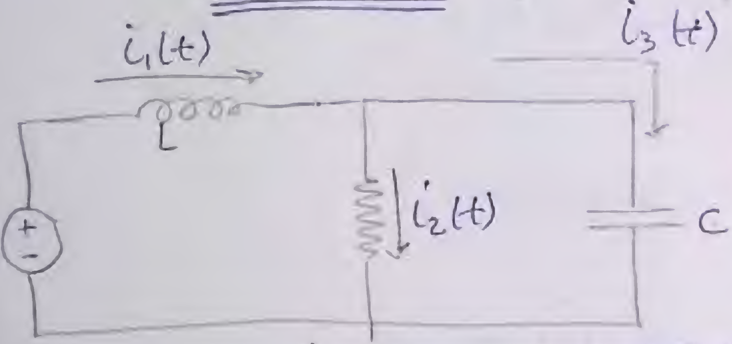
$y(t) = i_2(t) \implies y(t) = x_2(t)$

$$\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{pmatrix} = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & \frac{-1}{L_1} \\ 0 & \frac{-R_2}{L_2} & \frac{-1}{L_2} \\ \frac{1}{C} & \frac{-1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \cancel{D\,u(t)} \to 0$$

Report

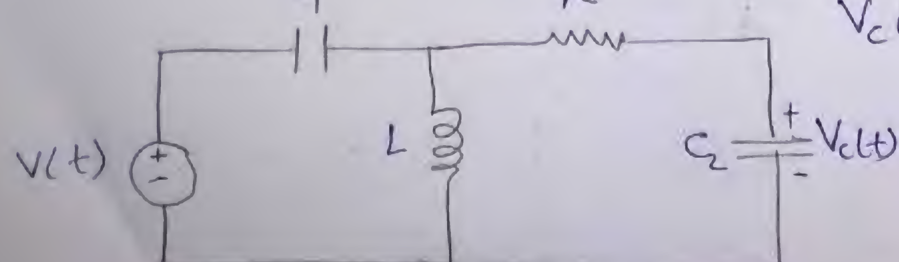
$\boxed{1}$



$i_2(t) \longrightarrow$ output

$\boxed{2}$

$V_c(t) \longrightarrow$ output



$\boxed{4}$ Lec18

$\boxed{ex}$

Given

$$G(s) = \frac{5s^2 + 8s + 15}{s^3 + 4s^2 + 10s + 20}$$

required

$$\dot{X}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) + D\,u(t)$$

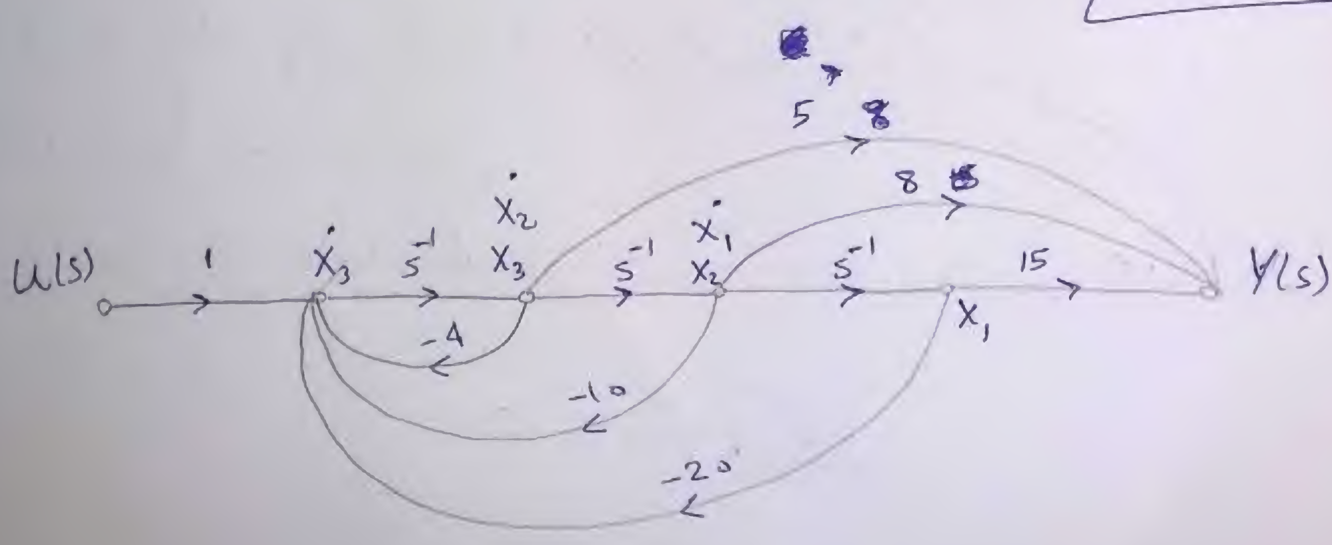
→ Draw signal flow graph :-

① نقسم البسط , المقام على أعلى درجة في المقام.

$$G(s) = \frac{5s^{-1} + 8s^{-2} + 15s^{-3}}{1 + 4s^{-1} + 10s^{-2} + 20s^{-3}} \Big\} \text{integral form}$$

$$= \frac{5s^{-1} + 8s^{-2} + 5s^{-3}}{1 - \left[-4s^{-1} - 10s^{-2} - 20s^{-3}\right]}$$

$\boxed{CCF}$



" state diagram

$$
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \overset{A_c}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -20 & -10 & -4 \end{bmatrix}} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \overset{B_c}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} u(t)
$$

$$
y(t) = \overset{C_c}{\begin{bmatrix} 15 & 8 & 5 \end{bmatrix}} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + D\, u(t)
$$

$$
G(s) = \frac{5s^2 + 8s + 15}{s^3 + 4s^2 + 10s + 20}
$$

← اذا طلب (ccf) تكتب المصفوفات مباشرة :-

A ← أول صف يكون (0 1 0) ثم ننقل "1" إلى اليمين (Right shift)

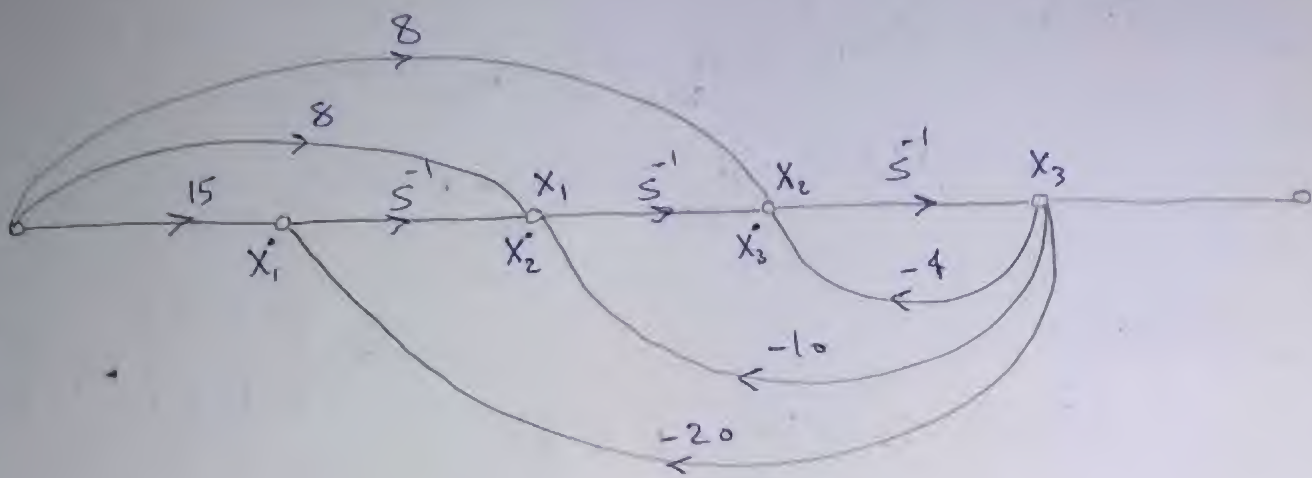
← آخر صف هو معاملات المقام بترتيب معكوس وإشارات معكوسة.

B ← كله صفر ما عدا آخر عنصر "1".

D ← يساوي صفر.

C ← معاملات البسط بترتيب عكسي بنفس الإشارات.

ـــ توجد طريقة أخرى تسمى (o.c.f)



$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -20 \\ 1 & 0 & -10 \\ 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 15 \\ 8 \\ 5 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + D\, u(t)$$

$A_c = A_o^T \qquad A_o = A_c^T$

$B_c = B_o^T \qquad B_o = C_o^T$

$C_c = B_o^T \qquad C_o = B_c^T$

Given $\longrightarrow$ $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{10}{s^2+3s+4}$

required $\dot{x}(t) = \underset{2\times2}{A}\ \underset{2\times1}{x(t)} + \underset{2\times1}{B}\ u(t)$

$y(t) = \underset{1\times2}{C}\ \underset{2\times1}{x(t)} + D\,u(t)$

Given $A = \begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\dot{x}(t) = A\,x(t) + B\,u(t)$

$y(t) = C\ x(t) + D\ u(t)$ $C = [10 \quad 0]$, $D = 0$

Required $G(s) = \dfrac{Y(s)}{U(s)} = C[sI - A]^{-1} B + D$

Sol

$$sI - A = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 4 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} s+3 & 1 \\ -4 & s \end{bmatrix} * \dfrac{1}{\begin{vmatrix} s & -1 \\ 4 & s+3 \end{vmatrix}}$$

$$= \begin{vmatrix} s+3 & 1 \\ -4 & s \end{vmatrix} * \dfrac{1}{s^2+3s+4}$$

$$C(sI-A)^{-1}B+D=\begin{bmatrix}1 & 0\end{bmatrix}\frac{1}{s^2+3s+4}\begin{bmatrix}s+3 & 1\\ 4 & s\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$=\frac{1}{s^2+3s+4}\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1\\ s\end{bmatrix}$$

$$G(s)=\frac{1}{s^2+3s+4}$$

Report

$$G(s)=\frac{X(s)}{U(s)}=\frac{8s^3+40s^2+50s+60}{s^4+s^3+20s^2+10s+30}$$

* Draw state diagram in c.c.f and o.c.f

* Draw state space in c.c.f and o.c.f